

## **An example Maths curriculum from the US.**

There is no National Curriculum in the US with each State setting it's own standards. The following is considered to be more academically rigorous than some.

Any 'grades' mentioned refer to [US grades](#)

### **Aged 5 - 6**

#### **Number Sense**

- count in various ways including counting objects up to 12, counting by ones up to thirty-one and backwards from ten, skip counting by fives and tens to 50 and by twos up to ten (2 to 10 and 1 to 9)
- identify written numbers from 0 to 31
- select the correct numeral to indicate a quantity from 0 to 9, trace over the numeral, and write the numeral
- select a reasonable order of magnitude from three given quantities -- a one-digit number, a two-digit number, and a three-digit number (e.g., 5, 50, and 500) -- for a familiar situation
- identify ordinal positions from first to fifth using concrete objects

#### **Number Facts - Addition and Subtraction**

- identify one more and one less for numbers from 1 to 9
- add and subtract whole numbers using up to 10 concrete items
- recognize and correctly use the + and - signs, and understand the meaning of adding to and taking away from

#### **Comparisons and Fractions**

- compare two sets of 10 or fewer concrete items to identify one as containing more, less, or the same as the other set.
- divide a set of 2, 4, 6, or 8 concrete objects into two equal halves

## **Measurement**

- identify the instruments used to measure time, length, weight and temperature
- make direct comparisons of objects according to length, weight, temperature and volume and measure lengths of objects using non-standard units of length (such as hand span, or new pencil length)

## **Time and Money**

- tell time to the hour using analogue and digital clocks
- sequence events in time (before vs. after, first vs. last)
- know the days of the week and the months of the year in order
- recognize a penny, nickel, dime, quarter and one dollar bill
- identify the dollar sign and cents sign, and write amounts to 9 cents using the cents sign

## **Patterns and Geometry**

- indicate the ordered position of each of three items in an ordered set from left-to-right, right-to-left, top-to-bottom, and bottom-to-top using both physical objects and pictures
- identify, describe, and make basic plane figures — square, rectangle, triangle, circle — and identify them in a variety of common objects, regardless of their orientation
- sort a set of objects based on one attribute (size, shape, colour, and quantity), identify the common property of the elements of a set, and identify the item that does not belong in a given set when all other items share a common property
- identify, describe, and extend a simple repeating pattern found in common objects and pictures (such as increasing size, alternating colours, etc)

## Aged 6 - 7

### Number Sense

- read and write numbers from 0 through 100
- count by ones, twos, fives, and tens from 0 to 100 and count objects in a given set containing up to 100 objects
- identify one dozen and one pair
- group concrete objects by ones and tens and recognize place values for ones, tens and hundreds
- identify the ordinal positions first through tenth using concrete objects and pictures

### Number Facts - Addition and Subtraction

- recall addition facts, sums to 12, and the corresponding subtraction facts
- complete addition and subtraction problems written both horizontally and vertically
- add 3 single digit numbers with pencil and paper
- add and subtract two-digit numbers without regrouping
- report one more, one less, ten more, and ten less from numbers from 10 to 90
- solve story and picture problems involving one-step solutions, using basic addition and subtraction facts
- solve simple addition and subtraction equations (to 12) with a blank in any position, such as  
 $2 + 5 = \underline{\quad}$ ,  $7 - \underline{\quad} = 5$ ,  $\underline{\quad} - 2 = 5$

### Comparisons and Fractions

- compare two sets of up to 12 objects, reporting the first to contain more or less than the second, and count the number more or less
- use the symbols  $<$ ,  $>$ , and  $=$  to compare two sets or pictures of sets of up to 12 objects and two numbers from 0 to 100
- identify one half, one third, and one fourth using concrete materials or pictures, and

divide concrete object sets to 12 into equal halves, thirds, and fourths

## **Measurement**

- estimate and measure length in inches and weight in pounds
- compare weights of objects using a balance scale
- measure and draw line segments in inches and centimetres
- estimate and measure volume in cups and identify a cup, a quart and a gallon
- compare the volumes of two given containers by using concrete materials (e.g., jelly beans, sand, water, and rice)
- associate temperature in degrees Fahrenheit with weather

## **Time and Money**

- know the days of the week and the months of the year, both in order and out of sequence
- tell time to the half-hour, using an analogue and digital clocks
- orient events in time: today using yesterday and tomorrow, morning and afternoon, this morning and yesterday morning, etc.
- compare duration of events as to taking more or less time
- recognize and use dollars and cents signs
- count and report the value of a set of pennies, nickels, or dimes whose total value is up to 100 cents
- identify the number of pennies equivalent to a nickel, a dime, and a quarter
- show different combinations of coins that equal the same amount of money

## **Patterns and Geometry**

- know and use terms of orientation and relative position, such as: closed/open, on/under/over, in front/in back (behind), between, in the middle of, next to, beside, inside/outside, around, far from/near, above/below, to the right of/to the left of, here/there

- sort concrete objects according to two attributes (such as color and shape)
- recognize, describe, and extend a wide variety of patterns, including size, color, shape, and quantity, including increasing, decreasing and repeating patterns with concrete materials and pictures
- identify the common property of the elements of a set (including function), select matching additions to the set, and identify the item that does not belong in a set
- identify, describe and sort basic solid figures: sphere, cube, cone
- draw and describe triangles, squares, rectangles, and circles according to number of sides, corners, and square corners
- describe objects in the environment as containing triangles, rectangles, squares, and circles

## **Graphing**

- interpret simple pictorial graphs.
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## Aged 7 – 8

### Number Sense

- count by twos, threes, fours and fives to 100
- count by hundreds and by fifties to 1,000
- count by tens from any given number
- count forward and backward in the range from 0 to 1,000
- count with tally marks in groups of 5
- read and write numbers from 0 to 1,000
- read and write numbers from 0 to 100 as words
- write two- and three-digit numbers in expanded form (such as writing  $500 + 60 + 7$  for 567)
- compare two whole numbers between 0 and 1,000, using symbols and words ( $>$ ,  $<$ , or  $=$ , "greater than," "less than," or "equal to")
- round to the nearest 10 for numbers from 0 to 100
- identify the ordinal positions first through twentieth
- identify odd and even numbers

### Number Facts - Addition and Subtraction

- recall basic addition facts, sums to 18 or less, and the corresponding subtraction facts
- add two numbers on paper to 999 without regrouping
- add three two-digit numbers on paper without regrouping
- estimate sums to 99 and the corresponding differences
- solve one-step addition and subtraction problems using data from simple charts and picture graphs
- solve basic word problems involving sums and differences to 12

- recognize and use the inverse relationship between addition and subtraction to solve problems such as  $4 + \underline{\quad} = 7$  and  $\underline{\quad} + 3 = 7$  and  $7 - \underline{\quad} = 3$
- identify one more, one less, ten more, ten less, one hundred more, and one hundred less than a given number (solution in the range 0 to 1,000)

## **Number Facts - Multiplication**

- recognize the multiplication sign, know what the terms factor and product mean in multiplication, and understand that multiplication represents repeated addition
- multiply single digit numbers by 0, 1, 2, and 10

## **Comparisons and Fractions**

- use the symbols  $<$ ,  $>$ , and  $=$  to compare two sets or pictures of sets of up to 12 objects and two numbers from 0 to 1,000
- identify the part of a set and/or region that represents one-half, one-third, one-fourth, one-eighth, and one-tenth and write the corresponding fraction

## **Measurement**

- estimate and make linear measurements to the nearest centimetre and inch, including the distance around a polygon (determine perimeter)
- make linear measurements in feet and inches, and in meters and centimetres
- know that one foot = 12 inches
- know abbreviations: ft, in, cm
- measure and draw line segments in inches to  $\frac{1}{2}$  inch and to one centimetre
- estimate and measure volumes in cups, pints, quarts, gallons and litres, compare these volumes using the concepts of more, less, and equivalent
- compare weights of objects using a balance scale

- estimate and measure weight in pounds and kilograms
- know abbreviations: lb, kg
- measure and record temperature in degrees Fahrenheit (to the nearest 2 degrees)

## **Time and Money**

- tell and write time to the quarter hour, using analog and digital clocks
- use a.m. and p.m.; noon and midnight
- solving simple problems on elapsed time
- using a calendar, identify the date, day of the week, month, and year
- write the date using words and numbers, and only numbers
- count, compare, and make change, using a collection of coins and one-dollar bills
- recognize relative value of penny, nickel, dime, quarter, and dollar
- read and write amounts of money using dollar and cents signs and the decimal point
- show different combinations of coins that equal the same amount of money

## **Patterns and Geometry**

- estimate and then count the number of square units needed to cover a given surface using grid paper
- estimate and then count the number of cubes in a rectangular box
- distinguish between square and rectangle as regards length of sides
- measure perimeters in inches of squares and rectangles
- identify solid figures: sphere, cube, pyramid, cone, cylinder and associate solid figures with planar shapes: sphere (circle), cube (square), pyramid (triangle)
- identify and describe a cube, rectangular solid, sphere, cylinder, and cone, according to the number and shape of faces, edges, bases, and corners.
- make congruent shapes and designs
- identify lines as horizontal, vertical, perpendicular, and parallel

- use names for lines and line segments (for example, line AB; segment CD)
- identify a line of symmetry and create simple symmetric figures using concrete materials.
- identify, create, and extend a wide variety of patterns using symbols and objects

## **Graphing**

- locate points from 1 to 10 on a number line
  - create and interpret simple bar graphs
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## **Aged 8 – 9**

### **Number Sense**

- read and write numbers from 0 to 999,999 with digits and words
- write numbers in expanded form to 999,999
- identify the place value for each digit up to the hundred-thousands
- compare two whole numbers between 0 and 999,999, using symbols ( $>$ ,  $<$ , or  $=$ ) and words ("greater than," "less than," or "equal to").
- round a whole number, 999 or less, to the nearest ten and hundred.
- identify ordinal positions from first to one-hundredth.
- read and write decimals to the hundredths

### **Number Facts - Addition and Subtraction**

- complete addition problems with and without regrouping (up to 10,000) of any two whole numbers, and the corresponding subtraction problems
- recall basic addition facts quickly (not just reconstruct them)
- mentally estimate a sum to 999 and the corresponding difference
- use mental computation strategies to simplify addition and subtraction problems
- recognize the addition of a negative number as the subtraction of a positive number.

### **Number Facts - Multiplication and Division**

- know multiplication facts to  $10 \times 10$
- multiply, by 10, 100, and 1,000 mentally
- multiply two whole numbers, with and without regrouping, in which one factor is 9 or less and the other is a multi-digit number up to three digits
- estimate a product to 1,000
- solve simple word problems involving multiplication.

- know the meaning of dividend, divisor, and quotient
- know basic division facts to 100 by 10
- know that you cannot divide by 0
- understand the equivalence of the different ways of writing division problems
- know that any number divided by 1 equals the original number
- divide two- and three-digit dividends by one-digit divisors
- identify the remainders in division problems
- understand multiplication and division as opposite operations, and use the inverse relationships between multiplication and division to solve problems such as  $8 \div \underline{\quad} = 2$ .
- check division by multiplying (and adding remainder)
- identify perfect squares to 100 and recognize the squared exponent
- solve two-step word problems
- solve equations in the form of  $\underline{\quad} \times 9 = 63$ ;  $81 \div \underline{\quad} = 9$
- solve problems with more than one operation, as in  $(43 - 32) \times (5 + 3) = \underline{\quad}$

### **Decimal Arithmetic**

- add and subtract with decimals expressed as tenths, using concrete materials and paper and pencil

### **Fractions and Mixed Numbers**

- identify fractions represented by drawings or concrete materials to ninths, and represent a given fraction using both concrete materials and symbols
- identify numerator and denominator
- write mixed numbers
- recognize equivalent fractions (for example,  $1/2 = 3/6$ )
- compare fractions with like denominators, using the signs  $<$ ,  $>$ , and  $=$
- compare the numerical value of two fractions having like and unlike denominators,

using concrete materials

- add and subtract with proper fractions having like denominators of 10 or less

## **Measurement**

- estimate and measure length in inches, feet, yards, centimetres, and meters
- know that one foot = 12 inches; one yard = 36 inches = 3 feet; 1 meter = 100 centimetres; 1 meter is a little more than one yard
- measure and draw line segments in inches (to  $\frac{1}{4}$  inch), and in centimetres (to \_ cm)
- estimate and measure liquid volume in cups, pints, quarts, gallons, and litres
- know that 1 quart = 2 pints; 1 gallon = 4 quarts
- compare a quart and a litre
- estimate and measure weight in pounds and ounces; grams and kilograms
- compare weights of objects using a balance scale
- know abbreviations: lb, oz, g, kg
- measure and record temperature in degrees Fahrenheit and Celsius
- know the degree sign
- identify freezing point of water in Fahrenheit and Celsius

## **Time and Money**

- identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours
- read a clock face and tell time to the minute, tell time in terms of both minutes before and minutes after the hour, and use a.m. and p.m.
- solve problems of elapsed time
- use a calendar, identify the date, day of the week, month, and year
- write the date using words and numbers, and only numbers
- determine by counting the value of a collection of bills and coins up to \$5.00,

compare the value of the coins or bills, and make change using as few coins as possible

- write amounts of money using dollar and cents signs, and the decimal point

## **Patterns and Geometry**

- know the term vertex (plural: vertices) and identify them
- identify sides of a polygon as line segments
- identify a regular pentagon, hexagon, and octagon
- identify right angles and know there are four in a square or rectangle
- compute area of rectangles in square inches and square centimetres using repeated addition and simple multiplication
- identify the shape of faces and edges in plane and solid geometric figures (square, rectangle, triangle, cube, rectangular solid, and cylinder)
- identify and draw representations of line segments and angles, using a ruler or straight-edge
- identify and describe congruent and symmetrical two-dimensional figures
- recognize and describe patterns formed using concrete objects, tables, and pictures and extend and reproduce the pattern

## **Graphing**

- locate zero, positive, and negative whole numbers on a number line
  - create and interpret simple line graphs
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## **Aged 9 – 10**

### **Number Sense**

- read and write numbers from -999,999,999 to 999,999,999.
- write numbers in expanded form to 999,999,999.
- identify, orally and in writing, the place value for each digit in a whole number expressed through hundred-millions
- compare two whole numbers between -999,999,999 and 999,999,999, using symbols ( $>$ ,  $<$ , or  $=$ ) and words ("greater than," "less than," or "equal to").
- round whole numbers to the nearest ten, hundred, and thousand.
- read, write, and identify decimals expressed through thousandths
- write decimals in expanded form
- identify place value of decimals to thousandths
- compare the value of two decimals through thousandths using the symbols  $>$ ,  $<$ , and  $=$
- round decimals to the nearest whole number, tenth, and hundredth

### **Decimal Arithmetic**

- add and subtract with decimals through thousandths
- solve problems involving making change in amounts up to \$100.00

### **Multiplication and Division, Multiples and Factors**

- multiply by two-digit and three-digit numbers
- solve word problems involving multiplication

- identify perfect squares (and square roots) to 144
- multiply mentally by 10, 100, 1,000, and 10,000
- use mental computation strategies for multiplication, such as breaking a problem into partial products, for example:  $3 \times 27 = (3 \times 20) + (3 \times 7) = 60 + 21 = 81$
- estimate and divide dividends up to four-digits by one-digit and two-digit divisors
- solve two-step word problems that include multiplication and division
- solve multiplication and division problems with money
- solve multiplication and division equations in the form of  $\_\_\_ \times 9 = 63$ ;  $81 \div \_\_\_ = 9$
- solve problems with more than one operation, as in  $(72 \div 9) \times (144 \div 12) = \_\_\_$
- identify multiples of a given number and common multiples of two given numbers
- identify factors of a given number and common factors of two given numbers

### **Fractions and Mixed Numbers**

- identify and write equivalent fractions and put fractions in lowest terms
- write mixed numbers and change improper fractions to mixed numbers
- rename fractions with unlike denominators to fractions with common denominators
- compare fractions with like and unlike denominators of 12 or less, using the signs  $<$ ,  $>$ , and  $=$
- add and subtract with fractions having like and unlike denominators of 12 or less

### **Relating Decimals and Fractions**

- read and write decimals as fractions (for example,  $0.39 = 39/100$ )
- relate fractions to decimals, using concrete objects

### **Measurement**

- estimate and measure length in parts of an inch ( $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$ ), inches, feet, yards, millimetres, centimetres, and meters
- estimate and measure liquid capacity in teaspoons, tablespoons, cups, pints, quarts, gallons, millilitres and litres
- estimate and measure weight in pounds and ounces, and in grams and kilograms
- know the following equivalences among U. S. customary units of measurement, and solve problems involving changing units of measurement: 1 ft = 12 in., 1 yd = 3 ft = 36 in., 1 mi = 5,280 ft, 1 mi = 1,760 yd, 1 lb = 16 oz, 1 ton = 2,000 lb., 1 cup = 8 fl oz, 1 pt = 2 c, 1 qt = 2 pt, 1 gal = 4 qt
- know the following equivalences among metric units of measurement, and solve problems involving changing units of measurement: 1 cm = 10 mm, 1 m = 1,000 mm, 1 m = 100 cm, 1 km = 1,000 m, 1 cg = 10 mg, 1 g = 1,000 mg, 1 g = 100 cg, 1 kg = 1,000 g, 1 cl = 10 ml, 1 litre = 1,000 ml, 1 litre = 100 cl
- estimate the conversion between ounces and grams, pounds and kilograms, inches and centimetres, yards and meters, miles and kilometres, and quarts and litres

## **Patterns and Geometry**

- identify and draw points, segments, rays, lines
- identify and draw lines -- horizontal, vertical, perpendicular, parallel, and intersecting — and angles — right, acute, and obtuse
- identify polygons — triangle, quadrilateral, pentagon, hexagon, octagon (regular), parallelogram, trapezoid, rectangle, square — and identify and draw diagonals of quadrilaterals
- identify the radius (plural: radii) and diameter of a circle and know that radius is half of the diameter
- recognize similar and congruent figures
- compute the area of a rectangle and solve problems involving finding area in a variety of square units (mi; yd; ft; in; km; m; cm; mm)
- compute volume of rectangular prisms in cubic units (cm, in)

- identify situations representing the use of perimeter and use measuring devices to find perimeter in both standard and non-standard units of measure
- extend a given pattern, using concrete materials and tables and solve problems involving pattern identification and completion of patterns

## **Graphing**

- read and write decimals on a number line
  - plot pairs of points on a coordinate grid using positive whole numbers
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# **Aged 10 – 11**

## **Number Sense**

- read, write, and identify the place values of decimals through ten-thousandths
- compare the value of two negative or positive decimals through ten-thousandths using the symbols  $>$ ,  $<$ , or  $=$
- write decimals in expanded form
- read and write decimals on a number line
- round decimals (and decimal quotients) to the nearest tenth; to the nearest hundredth; to the nearest thousandth

## **Multiplication and Division, Multiples and Factors**

- multiply two factors of up to four digits each
- know what it means for one number to be divisible by another
- divide dividends up to four-digits by one-digit, two-digit, and three-digit divisors
- move the decimal point when dividing by 10, 100, or 1,000
- solve division problems with remainders by rounding a decimal quotient
- identify prime numbers less than 50
- determine the greatest common factor and the least common multiple of given numbers

## **Decimal Arithmetic**

- estimate decimal sums, differences, and products by rounding
- add and subtract decimals through ten-thousandths

- estimate and find the product of two numbers expressed as decimals through thousandths
- estimate and find the quotient given a dividend expressed as a decimal through ten-thousandths and a whole number

## **Fractions**

- compare fractions with like and unlike denominators of 12 or less, using the signs  $<$ ,  $>$ , and  $=$
- determine the least common denominator (LCD) of fractions with unlike denominators
- compare fractions with like and unlike denominators, using the signs  $<$ ,  $>$ , and  $=$
- identify the reciprocal of a given fraction; know that the product of a given number and its reciprocal  $= 1$
- add and subtract with fractions and mixed numerals (with like and unlike denominators), with and without regrouping, and express answers in simplest form
- multiply mixed numbers and fractions
- write fractions as decimals (e.g.,  $1/4 = 0.25$ ;  $17/25 = 0.68$ ;  $1/3 = 0.3333 \dots$  or  $0.33$ , rounded to the nearest hundredth)

## **Ratios and Percent**

- determine and express simple ratios
- use ratio to create a simple scale drawing
- solve problems on speed as a ratio, using the formula  $S = d / t$  (or  $D = r \times t$ )
- recognize the percent sign and understand percent as per hundred
- find the given percent of a number
- express equivalences between fractions, decimals, and percent, and know the percentage equivalent for  $1/10$ ,  $1/4$ ,  $1/2$ , and  $3/4$

## **Measurement**

- estimate and make linear measurements in yards, in feet and inches (to  $\frac{1}{16}$  in.), and in meters, centimetres, and millimetres
- convert to common units of measurement in problems involving addition and subtraction of different units
- choose an appropriate measuring device and unit of measure to solve problems involving measurement of length in parts of an inch, inches, feet, yards, miles, millimetres, centimetres, meters, and kilometres; weight/mass in ounces, pounds, tons, grams, and kilograms; liquid volume in cups, pints, quarts, gallons, millilitres, and litres; area in square units of length; and temperature in degrees Celsius and Fahrenheit
- estimating the conversion between Celsius and Fahrenheit
- determine an amount of elapsed time in hours and minutes to 24 hours, including crossing noon or midnight

## **Geometry**

- determine the perimeter of a polygon and the area of a square, rectangle, and triangle, given the appropriate measures
- identify the diameter, radius, chord, and circumference of a circle
- differentiate between area and perimeter and identify whether the application of the concept of perimeter or area is appropriate for a given problem
- measure angles in degrees and know the meaning of right angle, acute angle, obtuse angle, and straight angle
- identify and construct different kinds of triangles -- equilateral, right, and isosceles
- define what it means for triangles to be congruent
- know that regular polygons have sides of equal length and angles of equal measure
- identify and draw diagonals of polygons

- work with circles to identify arc, chord, radius and diameter
- use a compass, draw circles with a given diameter or radius
- find the circumference of a circle using the formulas  $C = \pi d$ , and  $C = 2 \pi r$ , using 3.14 as the value of  $\pi$
- find the area of a rectangle, triangle, and parallelogram in a variety of square units (mi, yd, ft, in, km, m, cm, mm)
- find the area of an irregular polygon by dividing it into regular figures
- compute volume and surface area of a rectangular prism
- describe and extend numerical and geometric patterns, including triangular numbers, perfect squares, patterns formed by powers of 10, and arithmetic sequences

## **Algebra and Graphing**

- know the names and of the commutative and associative properties for addition, and the commutative, associative, and distributive properties for multiplication, and illustrate understanding by usage and identifying examples and counter examples
  - recognize variables and solve one-operation equations using variables
  - write and solve equations for word problems using variables
  - identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane
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## **Aged 11 – 12**

### **Number Sense**

- read, write, and order positive and negative decimals to the nearest hundred-thousandth
- write decimals in expanded form and write numbers in expanded form with scientific notation
- round whole numbers to the nearest ten through million
- round decimals (and decimal quotients) to the nearest whole number, tenth, hundredth, and thousandth
- read and evaluate numerical expressions with exponents
- identify powers of 10 to  $10^6$
- compare positive and negative decimals, mixed numbers, whole numbers and fractions with like and unlike denominators, using the signs  $<$ ,  $>$ , and  $=$ , including scientific notation.

### **Decimals, Fractions, Ratios and Percents**

- estimate decimal sums, differences, products and quotients with rounding, and verify the solution
- determine whether a number is a prime number or a composite number, and explain the concepts of prime and composite numbers
- identify the reciprocal of a given fraction and know that the product of a given number and its reciprocal = 1
- round fractions to the nearest whole number,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{8}$ , and  $\frac{1}{10}$
- translate among percent, fractions and decimals, including repeating decimals
- add and subtract positive and negative decimals, mixed numbers, whole numbers

and fractions with like and unlike denominators

- multiply and divide positive and negative decimals, mixed numbers, whole numbers and fractions, including dividing by a fraction
- solve problems involving percent increase and decrease and with percent greater than 100%
- solve problems that involve addition, subtraction, and/or multiplication with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators, and express their answers in simplest form
- use estimation strategies to solve multi-step practical problems involving whole numbers, decimals, and fractions.
- compare two values or variables as ratios using appropriate notations such as  $a/b$ ,  $a$  to  $b$ , and  $a:b$
- solve proportions, including word problems involving proportions with one unknown
- use ratios and proportions to interpret map scales and scale drawings
- solve multi-step consumer application problems involving fractions and decimals
- recognize probability as a measure of the likelihood that an event will happen and express probability of a given event as a fraction and as a ratio

## **Measurement**

- Associate prefixes used in metric system with quantities: kilo, hecto, deka, deci, centi, milli
- compare and convert units of measures for length, weight/mass, and volume within the U.S. Customary system and within the metric system and estimate conversions between units in each system

## **Geometry**

- estimate angle measures to 30 degrees and use the appropriate tools to measure the given angles

- identify and use signs that mean is congruent to, is similar to, is parallel to, and is perpendicular to
- construct parallel lines and a parallelogram
- know that, if two lines are parallel, any line perpendicular to one is also perpendicular to the other
- know that two lines that are both perpendicular to another line are parallel to each other
- bisect an angle
- construct an angle congruent to a given angle
- construct a figure congruent to a given figure, using reflection over a line of symmetry, and identify corresponding parts
- Show how congruent plane figures can be made to correspond through reflection, rotation, and translation
- know that sum of the measures of the angles of a triangle
- identify congruent angles and sides, and axes of symmetry, in parallelograms, rhombuses, rectangles, and squares
- find the area and perimeter of a rectangle, square, triangle, parallelogram, and circle
- find the volume of rectangular solids and find a missing dimension given the volume
- determine if geometric figures (quadrilaterals and triangles) are similar and write proportions to express the relationships between corresponding parts of similar figures

## **Algebra and Graphing**

- Recognize variables and solve linear equations in one variable
- write and solve equations for word problems
- create data summaries in graphic form (bar, line, and circle graphs)
- solve problems requiring interpretation and application of graphically displayed data

- plot points on a coordinate plane, using ordered pairs of positive and negative whole numbers
  - use the terms origin, x-axis, and y-axis working with the coordinate plane
  - graph simple functions and solve problems involving use of a coordinate plane
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## Pre-Algebra

Students in seventh, eighth or ninth grades who have not taken Algebra are expected to be preparing for Algebra. Various different sources indicate that there are certain essential skills and aspects of mathematical knowledge that a student must master in order to succeed in Algebra. Of particular importance are operations with fractions, decimals and percents, operations with integers and operations using negative as well as positive numbers. Failure to master these makes it extremely unlikely that a student will thrive in Algebra.

These Standards list skills that students should master. They need not be taught in the order presented. Some topics may appear in slightly different forms in different areas. These Standards mention a number of specific skills as well as solving "word" or "real world" problems. Even when not mention explicitly, students should practice and be able to use the component skills in the context of solving "word" or "real world" problems.

Some of these Standards will have been met by students in an appropriate K-6 program. If not, students expecting to take Algebra are expected to meet these Standards in their Pre-Algebra course. For some students, such a course might be designed to take two years. A number of different textbooks should be able to prepare students to meet these Standards, but textbooks that are clearly lacking in large portions of these Standards should not be used for courses that serve as the course before Algebra. Although some textbooks or classroom methods may involve calculators as pedagogical aids, it is expected that a student will demonstrate mastery of the material described in these Standards without the use of a calculator.

### Properties of the Number System

PA-1 The student will know and identify the following properties or operations with real numbers and use them to justify individual steps in the solution of problems:

- the commutative and associative properties for addition and multiplication;
- the distributive property;
- the additive and multiplicative identity properties;
- the additive and multiplicative inverse properties; and
- the multiplicative property of zero.

- PA-2 The student will be able to perform operations with exponents, including operations with positive, negative and fractional exponents. The student should also be able to manipulate numbers expressed as powers of 10 and in scientific notation.
- PA-3 The student will break numbers into their prime factors.
- PA-4 The student will know the squares of numbers up to 16 and be able to determine the value, to between any two integers, of the square root of any number less than 256.
- PA-5 The student will know and use the rules for order of operations to evaluate numerical expressions, to evaluate algebraic expressions in which the variables are replaced by specific values, to simplify algebraic expressions containing up to three variables and to solve linear algebraic equations of a single variable.

### **Fractions, Decimals, Percents, Proportions, Ratios and Probability**

- PA-6 The student will add, subtract, multiply and divide fractions, including fractions with unlike denominators and negative fractions.
- PA-7 The student will add, subtract, multiply and divide decimals, and decimals expressed as powers of ten or in scientific notation.
- PA-8 The student will convert fractions to decimals, decimals to fractions, fractions to percents, percents to fractions, decimals to percents, and percents to decimals.
- PA-9 The student will solve "word" or "real world" problems using fractions, decimals and percents, including problems involving determining what fraction or percent of one quantity another quantity is, or determining what value is a set fraction or percent of a given quantity. This will include problems involving money such as computation of tips, discounts, sales tax, and simple interest.
- PA-10 The student will recognize and compute proportions and ratios as a fraction of a total and in addition, will calculate ratios as the relationship of two parts of a total.
- PA-11 The student will solve "word" or "real world" problems involving proportions and ratios.
- PA-12 The student will use proportions/ratios to evaluate scale drawings and to produce scale drawings.

- PA-13 The student will recognize the relationship between probability statements and fractions, decimals and percents, will determine the expected number of events based on a given probability, and will be able to estimate the probability of an event based on a given distribution of previous events.
- PA-14 The student will determine the theoretical probability of events in simple systems such as the chance of drawing a "heart" or an "ace" from the a deck of cards, the chance of rolling a particular number with one die or a pair of dice, or the chance of a particular result on a spinner.

### **Algebraic Manipulations**

- PA-15 The student will evaluate expressions in one, two or three variables when given specific numerical values for each variable using any or all of the processes covered above.
- PA-16 The student will simplify expressions involving one, two or three variables such as
  - $m^3p + mpm^2 - m^3 + 3pmp^2$  or
  - $a^2bc/ab^3$
- PA-17 The student will solve linear equations and inequalities in one variable such as
  - $2x/5 = -10$  or
  - $5y - 2.7 = -12.7y$  or
  - $3r - 6/5 = 9/3$
- PA-18 The student will, given various "word" or "real world" problems, write and solve linear equations of a single variable and convert these solutions into answers to the problems.

### **Graphing of Data and Equations**

- PA-19 The student will understand the terms domain, range, x-axis, y-axis, x-coordinate, y-coordinate, slope, x-intercept and y-intercept.
- PA-20 The student will plot ordered pairs of points in all quadrants of the coordinate plane.
- PA-21 The student will plot data in various graphical formats and will answer

questions and make predictions based on information given in graphs.

- PA-22 The student will calculate the mean, median, mode and range of a set of data.

## **Geometry**

- PA-24 The student will calculate the area and perimeter of rectangles, parallelograms, trapezoids, triangles and circles. The student will also calculate the area of more complicated shapes by breaking them into pieces such as rectangles, triangles and fractions of circles.
  - PA-25 The student will calculate the volume and surface area of right angle boxes and spheres. The student will also calculate the volume and surface area of rectangular prisms with known heights and irregular bases.
  - PA-26 The student will use the Pythagorean Theorem to calculate the length of the missing side of a right triangle and to determine if a triangle is a right triangle.
  - PA-27 The student will recognize and name polygons with up to 10 sides as well as dodecagons.
  - PA-28 The student will recognize congruent and similar polygons and will use proportions to determine the lengths of unlisted sides.
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# Algebra I

The standards for Algebra I are based on entering students having mastered the material covered in the Pre-Algebra Standards. This includes, specifically, mastery of the manipulation and interconversion among fractions, decimals and percents; mastery of the operations of arithmetic with negative as well as positive numbers; and mastery of the use and manipulation of exponents and radicals as applied to expressions involving integers.

In the course of meeting these standards, students will demonstrate substantial growth in their ability to solve problems using multiple algebraic methods. This includes expansion in the kind and complexity of word sentences a student can translate into mathematical expressions; expansion of the kind and difficulty of expressions a student can manipulate and solve; use of some techniques of analytic geometry; recognition of the possible use of multiple different methods to generate precise or approximate solutions to problems; and a recognition of the relative strength and weaknesses of different strategies as applied to specific problems.

- A1-1 The student will translate between problem situations, verbal expressions, and mathematical expressions with variables, and extend this process to include problems involving exponents and simple radicals, polynomials, absolute values and inequalities. For expressions that are not written as equations, the student will evaluate these expressions for given replacement values of the variables. For equations or sets of equations, the student will determine the values of the variables that constitute the solution set of the equation or set of equations.
- A1-2 The student will solve linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable and apply these skills to solve practical problems.
- A1-3 The student will extend the properties of real numbers into the context of algebraic equations with variables and be able to explain algebraic manipulations in terms of the properties of real numbers. In this way, the student will justify each step used in the process of simplifying expressions and solving equations and inequalities.
- A1-4 The student will translate freely among various representations of linear

equations, including the slope of the line and a point on it; two points on a line; a point on the line and the condition that the line is parallel or perpendicular to another given line; a graph of the line or of points on it; a problem situation or word problem representing a line; and an equation for the line in slope-intercept form, standard form, or arbitrary form.

- A1-5 The student will translate an equation for a line in any form to an equation for either variable in terms of the other variable, and use the equation to find values for one variable given replacement values for the other variable. This includes using a linear equation in a problem situation to solve the problem, and recognizing when a linear equation cannot be used to solve a problem.
- A1-6 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. The student will also describe the slope as a rate of change and identify slopes as positive, negative, zero, or undefined
- A1-7 The student will solve systems of two linear equations in two variables by graphical estimation and by algebraic techniques including substitution and the addition and subtraction of equations (with and without a multiplication step). These techniques will be applied to solve practical problems. Students will extend this work to graphing the solution set of two linear inequalities. Students will also solve systems of addition and subtraction equations in three variables with three unknowns by substitution.
- A1-8 The student will use the Pythagorean Theorem and its converse to find distance measures in the special case of right triangles, and use the representation of this theorem in the coordinate plane, the distance formula, to find the distance between any two points or the length of a specified line segment between two points. The student will also explain each step when given a proof of the Pythagorean Theorem.
- A1-9 The student will determine the domain and range of a relation given a set of ordered pairs, a graph, or a function rule, and will identify the relations that are and are not functions.
- A1-10 The student will draw mapping diagrams for ordered pairs and vice versa, graph functions and relations over finite domains in the coordinate plane, and, given a function rule, find the values of a function for elements in its domain and locate

the zeros of the function algebraically.

- A1-11 The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial and consumer situations.
- A1-12 The student will factor completely binomials and trinomials, including quadratics and expressions having coefficients for the highest order term greater than one, in one or two variables when they are factorable over the rational numbers.
- A1-13 The student will factor special forms of (factorable) polynomials, including those requiring regrouping or repeated factorization and those having coefficients for the highest order term greater than one. This will include the extraction of monomial and binomial factors from expressions in the third or fourth degree.
- A1-14 The student will simplify expressions involving radicals, including the square roots of expressions involving constants and variables, and expressions involving the sum, difference, and products of radicals of both real numbers and monomial and binomial expressions, and use radical expressions to solve problems.
- A1-15 The student will add, subtract, and multiply polynomials and divide polynomials with monomial and binomial divisors. The student will also simplify rational algebraic expressions by combining like terms and by addition, subtraction, multiplication, and division of the polynomial components of these expressions.
- A1-16 The student will graph quadratic equations and inequalities. The student will estimate solutions to quadratic equations in one variable graphically and solve these equations algebraically by factoring and by using the quadratic formula. These techniques will be applied to the solution of problems involving quadratics.
- A1-17 The student will analyse a given set of data for the existence of a pattern, represent the pattern algebraically and graphically, if possible, and determine if the relation is a function.
- A1-18 The student will analyse a relation to determine whether a direct or inverse variation exists and represent it algebraically and graphically if possible.
- A1-19 The student will, given a set of data points, write an equation for a line of best fit, using the median fit method, and use the equation to make predictions.
- A1-20 The student will compare multiple one-variable data sets, using statistical

techniques that include measures of central tendency, range, stem and leaf plots, and box and whisker graphs.

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# Geometry

This course is designed for students who have successfully completed the standards for Algebra I. The course, among other things, includes the deductive axiomatic method of proof to justify theorems, to identify logical errors in faulty proofs and to tell whether conclusions are valid. Methods of justification will include paragraph proofs, flow charts, two-column proofs, indirect proofs, coordinate proofs, and verbal arguments.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques should be used to implement these standards, including algebraic skills.

- G.1 The student will recognize the three undefined terms "point", "line", and "plane", and their symbols, and be able to use these to define other terms such as "space", "ray", "angle" and so on.
- G.2 The student will know the five major "existence" postulates about points lines and planes:
  - 1) A line contains at least two points, a plane contains at least three points not all on one line; a space contains at least four points not all on one plane;
  - 2) There is exactly one line through two points;
  - 3) There is exactly one plane through three points not on one line;
  - 4) If two points lie in a plane, then the line joining them lies in that plane;
  - 5) If two planes intersect, their intersection is a line.
- G.3 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
  - identifying the converse, inverse, and contrapositive of a conditional statement;
  - translating a short verbal argument into symbolic form;
  - diagramming arguments involving quantifiers (all, no, none, some), using Venn diagrams;
  - using valid forms of deductive reasoning, including the law of syllogism; and
  - recognizing logical errors in faulty arguments.

- G.4 The student will use pictorial representations and coordinate methods to solve problems involving symmetry and transformation. This will include
  - using formulas for finding distance, midpoint, and slope;
  - investigating and determining whether a figure is symmetric with respect to a line or point; and
  - determining whether a figure has been translated, reflected, or rotated.
- G.5 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons. The student will know and use the Exterior Angle Theorem to find angle measures in triangles.
- G.6 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.
- G.7 The student will
  - identify congruence and similarity relationships between triangles; and
  - prove two triangles are congruent or similar given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.
- G.8 The student will be able to state and be able to use the Triangle Inequality Theorem. Given information concerning the lengths of sides and/or measures of angles, the student will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.
- G.9 The student will solve practical problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. Because special right triangles and their properties recur, the student will commit to memory the side ratios of special right triangles and use them to solve triangle problems.
- G.10 The student will
  - identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
  - prove these properties of quadrilaterals using algebraic and coordinate as

well as deductive proofs;

- use properties of quadrilaterals to solve practical problems.
- G.11 The student will understand the meaning of the term "regular polygon" and be able to determine the measures of the interior and exterior angles of regular polygons. In addition, the student will use measures of interior and exterior angles of polygons to solve problems.
- G.12 The student will use the properties of angles, arcs, chords, tangents, and secants to solve problems involving circles. Problems will include finding the area of a sector and doing constructions. At the honors level, this will include constructing inscribed or circumscribed circles given a triangle; locating the center of a circle; and constructing the tangent to a circle from a point on the circle and from a point not on the circle.
- G.13 The student will construct, using a compass and straightedge, a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, an angle congruent to a given angle, and a line parallel to a given line from a point not on the line.
- G.14 The student will
  - understand the meaning of 'locus' and be able to describe and draw the locus of points satisfying a given condition;
  - solve locus problems using constructions. In particular he or she will be able to locate the circumcenter, incenter, orthocenter and centroid of a given triangle using constructions previously learned.
- G.15 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems.
- G.16 The student, given similar geometric objects, will use proportional reasoning to solve practical problems; investigate relationships between linear, square, and cubic measures; and describe how changes in one of the measures of the object affect the others.